Computational Geometry—236719

(Fall 2023–2024, Gill Barequet and Tomer Adar)

Assignment no. **1**

Given: 30/01/2024
Due: **13/02/2024**

Submission in **singletons**

Question 1.

A set $S \subset \mathbb{R}^2$ (or in any dimension) is *convex* if for every two points $p, q \in S$, the line segment pq is entirely contained in S. A set $S \subset \mathbb{R}^2$ is *star-shaped* if there exists a point $c \in S$ such that for every point $p \in S$, the line segment cp is contained in S. Prove or disprove:

(a) The intersection of two convex sets is convex.

(b) The union of two convex sets is star-shaped.

(c) The intersection of two star-shaped sets is star-shaped.

(d) The intersection of a convex set and a star-shaped set is convex.

Question 2.

Let S be a set of n circles in the plane. Describe a plane-sweep algorithm which computes all the intersection points of the circles. The algorithm should run in $O((n+k)\log n)$ time, where k is the number of intersection points.

Question 3.

(a) In a DCEL, which of the following equalities are always true?
- $Twin(Twin(e)) = e$
- $Next(Prev(e)) = e$
- $Twin(Prev(Twin(e))) = Next(e)$
- $IncidentFace(e) = IncidentFace(Next(e))$

(b) Give a pseudocode for the following algorithms using a DCEL subdivision:
- List all vertices that are connected by an edge to a given vertex v.
- List all edges that bound a given face f in a not necessarily connected subdivision.
- List all faces that have at least one vertex on the outer boundary of the subdivision.

(c) Given a doubly-connected edge list representation of a subdivision where $Twin(e) = Next(e)$ holds for every half-edge e, how many faces can the subdivision have at most?

Question 4.

(a) Give an efficient algorithm to determine whether or not a polygon P with n vertices is monotone with respect to a given line ℓ (not necessarily horizontal or vertical).

(b) Prove or disprove: The dual graph of any trianglation of a monotone polygon is always a chain, that is, any node in this graph has degree at most two.

Question 5.

(a) Prove that any simple polygon, even if it has holes (which are also simple polygons), has a triangulation.

(b) Let P be a simple polygon with h simple polygonal holes, and n vertices in **total**. What is the number of triangles in a triangulation of P? Prove your answer.

(c) What is T_n, the number of different triangulations of a convex polygon with n vertices? Express T_n in a recursive manner, that is, in terms of $T_1, \ldots, T_{n-1}$.