Computational Geometry—236719

(Fall 2023–2024, Gill Barequet and Tomer Adar)

Assignment no. **3**

Given: March 18, 2024
Due: **April 1, 2024**

Submission in **singletons**

Question 1.
Let L be a set of n lines in the plane. Give an $O(n \log n)$-time algorithm for computing an axis-parallel rectangle that contains all the vertices of $A(L)$ in its interior.

Question 2.

1. Let $S = \{p_1, ..., p_n\}$ (for $n \geq 3$) be the vertices of a regular convex polygon, and let C be its center. Let $P = S \cup C$. Prove that in the Voronoi diagram of P, the Voronoi cell of C contains n vertices. (A regular polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length).
2. Assuming general position, prove that for a Voronoi diagram of n points, the average number of vertices of a cell is arbitrarily close to 6 as $n \to \infty$.

Question 3.
GG(S), the *Gabriel Graph* of a point set S in the plane, is defined as follows: Two points $p, q \in S$ are connected by an edge of the graph if the circle with diameter pq does not contain any other point of S in its interior.

1. Prove that DT(S) (Delaunay Triangulation of S) contains the Gabriel graph of S.
2. Prove that p and q are adjacent in GG(S) iff the Delaunay edge that connects between them intersects its dual Voronoi edge.
3. Give an $O(n \log n)$-time algorithm to compute the Gabriel graph of a set of n points.

Question 4.
Let S be a set of n points in the plane, and let t be the number of lines that pass through **exactly** $\sqrt{n}$ points of S. Using the crossing-number lemma, prove that $t = O(\sqrt{n})$.